MICROWAVE HOLOGRAM RECONSTRUCTION FOR THE RASCAN TYPE SUBSURFACE RADAR

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ABSTRACT

In this paper the mathematical models and results on processing the experimental single-frequency microwave holograms received by scanning subsurface radar with sine wave signal are submitted. The holograms reconstruction method with the use of support functions, which take into account the near field of the aperture antenna with round cylindrical waveguide, is analysed. The models consider both known and unknown phase shift of the signal reflected from the point object. It is theoretically and experimentally shown that single-frequency holograms reconstruction allows to estimate depth of shallowly buried objects and improve the resolution on the probing surface with the growth of objects depths.

Key words: Subsurface radar, Microwave hologram, Reconstruction algorithm, Near and reactive fields.

INTRODUCTION

Subsurface holographic radar of the RASCAN type uses continuous wave unmodulated signals, which are transmitted in single-frequency or multi-frequency modes (Vasiliev et al., 1999). In the radar antenna, signal reflected from the object mixes with reference one. To obtain holographic images the radar antenna scans two-dimensionally along interface of lossy half-space. The amplitudes of received signal are recorded in discrete set of points at the surface (x, y, $z=0$), Fig. 1. Measured amplitudes of the radar signal are displayed as microwave holograms on computer screen.

The antenna with cylindrical waveguide, which is used in the radar, has about one and a half wavelength diameter, which corresponds to 6...7 cm for frequency range of 3.6...4.0 GHz. The antenna operates as the gauge of the reflected electromagnetic (EM) waves in the near and reactive fields for objects depths from zero level up to 20...25 cm.

Since the images recorded by radar are microwave holograms, it is impossible to determine the objects depths under the scanning surface without the appropriate processing. Moreover horizontal resolution is deteriorated with the increase of depth. This effect is typical for all types of subsurface radars (Daniels, 1996).

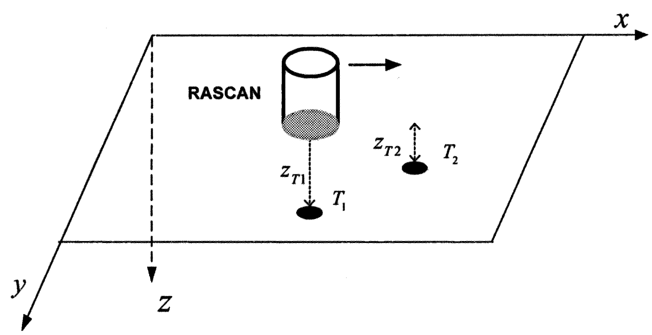

Figure 1. 2D data acquisition using the RASCAN subsurface radar.

In this paper interpretation of recording and reconstruction of the microwave holograms for subsurface radar of the RASCAN type is given. As an approximation, the scalar model of the field emitted by the aperture antenna is chosen. The hologram reconstruction, executed with the help of the developed theoretical models, enables to improve the horizontal resolution with the increase of objects depths and estimate their depths.

The essential feature of the models, in comparison with earlier works (Junkin, and Anderson, 1988; Popov, Kopeikin, and Vinogradov, 2000), is that registration of holograms by the RASCAN radar is carried out in the near and reactive fields of the real antenna aperture. In the analysis that follows, we shall consider monochromatic microwave holograms reconstruction for two variants of signal registration:

a) The case of the complex amplitude-phase hologram recorded by the receiver with two quadrature channels.
b) The case of the amplitude hologram recorded with the use of amplitude receiver.

In both cases the presence of the constant leakage (reference) signal of transmitter with the unknown phase difference in relation to the phase of a scattered object wave is taken into account.

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MATHEMATICAL MODEL OF THE SYNTHESIZED MICROWAVE HOLOGRAM FOR POINT OBJECTS

For the analysis of microwave holograms received by RASCAN subsurface radar on the surface $(x, y, z=0)$ we consider model of observation of a point object at the depth $z=z_T$ in lossy half-space. By a point object we mean the object with dimensions much smaller than the wavelength. Let's consider, that the waveguide diameter of the antenna is equal to D.

Analyzing real and imaginary components of the complex hologram, we proceed from the scalar Fresnel-Kirchhoff integral equation (Skolnik, 1970; Papylis 1971) for the near and reactive fields of the antenna equitable for one component of the EM field vector. At the uniform amplitude and phase excitation of the circular aperture the transformation from Cartesian to polar coordinates in the Fresnel-Kirchhoff integral equation gives us the complex amplitude of the field as follows

$$\begin{aligned}\dot{E}(\rho,z) = &\frac{j\dot{k}}{2\pi} \\ &\times \int_0^{D/2} \delta \int_0^{\pi} \frac{\exp\left\{-j\dot{k}\sqrt{\rho^2+\delta^2-2\rho\delta\cos\psi+z^2}\right\}}{\sqrt{\rho^2+\delta^2-2\rho\delta\cos\psi+z^2}} \\ &\times [1+\frac{z}{\sqrt{\rho^2+\delta^2-2\rho\delta\cos\psi+z^2}} \\ &\times (1+\frac{1}{j\dot{k}\sqrt{\rho^2+\delta^2-2\rho\delta\cos\psi+z^2}})]d\psi d\delta,\end{aligned} \tag{1}$$

where $\rho=\sqrt{x^2+y^2}$; $\dot{k}=\beta-j\alpha=(2\pi/\lambda_0)\cdot\sqrt{\varepsilon_1[1-j\sigma/(\varepsilon_0\varepsilon_1\omega)]}$ is the complex wave number of the lossy medium with relative permittivity ε_1 and conductivity σ; ε_0 is absolute permittivity of free space; ω and λ_0 are circular frequency and wavelength of EM field in vacuum accordingly. As far as the same aperture antenna performs reception and transmission of a signal, the received EM wave is proportional to the square of eqn. 1

$$\dot{E}^2(\rho,z) = A(\rho,z)\exp[j\Phi(\rho,z)] \ . \tag{2}$$

The substitution of variables $\rho=\sqrt{x^2+y^2}$ in eqn. 2 allows to find the complex two-dimensional hologram of the point object located at the depth z in the point $(0, 0, z)$.

For several point objects with coordinates (x_{Ti}, y_{Ti}, z_{Ti}) the complex microwave hologram is expressed as

$$\begin{aligned}\dot{H}(x,y) = &\dot{U}_0 + \dot{U}(x,y) = a_0\exp(j\varphi_0) \\ &+\sum_{i=1}^{N} a_{Ti} A\left(\sqrt{(x-x_{Ti})^2+(y-y_{Ti})^2}\,;z_{Ti}\right) \\ &\times\exp\left\{j\Phi\left(\sqrt{(x-x_{Ti})^2+(y-y_{Ti})^2}\,;z_{Ti}\right)+j\varphi_{Ti}\right\}+\dot{N}(x,y) \ ,\end{aligned} \tag{3}$$

where N is the number of objects; $\dot{U}_0$ is the reference (or leakage) signal with constant phase and amplitude; $\dot{U}(x,y)$ is the variable component of the complex hologram; a_0, φ_0 are the amplitude and the phase of the reference signal; a_{Ti} and φ_{Ti} are the amplitude and the unknown phases of the signals reflected from the objects; $\dot{N}(x,y)$ is the noise function; $A(\rho,z)$ and $\Phi(\rho,z)$ are functions of z and ρ, which are tabulated with the help of eqns. 1 and 2 (for the considered medium characteristics, diameter of the aperture D and frequency f).

In the case of the coherent quadrature reception device, the complex hologram $\dot{H}(x,y)$ is observed. For the single object $N=1$ and in absence of noise the microwave hologram after compensation of a reference signal $\dot{U}_0$ is represented as

$$\begin{aligned}\dot{H}(x,y) = &a_T A\left(\sqrt{(x-x_T)^2+(y-y_T)^2}\,;z_T\right) \\ &\times\exp\left\{j\Phi\left(\sqrt{(x-x_T)^2+(y-y_T)^2}\,;z_T\right)+j\varphi_T\right\} \ ,\end{aligned} \tag{4}$$

where φ_T is the unknown phase of the reflected signal from the object.

For case of using the amplitude receiver, the square of the complex hologram modulus is observed. In absence of noise it can be described as follows

$$\begin{aligned}h(x,y) = &\left|\dot{U}_0+\dot{U}(x,y)\right|^2 = \\ &\left|\dot{U}_0\right|^2+\left|\dot{U}(x,y)\right|^2+2\,\mathrm{Re}\left\{\dot{U}_0^{\,*}\cdot\dot{U}(x,y)\right\} \ .\end{aligned} \tag{5}$$

The basic information of the amplitude hologram (eqn. 5) is represented in the third summand, which has thin oscillatory structure. The second summand corresponds to a square of the envelope of these oscillations in the changed scale. In the case of the sole object under the surface and at conditions of $a_T/a_0 << 1$, deletion of the constant component, and absence of noise, eqn. 5 can be written as

$$\begin{aligned}h(x,y) \approx &2a_T\,a_0\,A\left(\sqrt{(x-x_T)^2+(y-y_T)^2}\,;z_T\right) \\ &\times\cos\left\{\Phi\left(\sqrt{(x-x_T)^2+(y-y_T)^2}\,;z_T\right)+\varphi\right\} \ ,\end{aligned} \tag{6}$$

where $\varphi=\varphi_T-\varphi_0$ is the unknown phase shift of the object with regard to the reference signal phase.

For single-frequency case, Fig. 2 shows the result of the calculation of the synthesized complex hologram quadrature components according to the proposed model for the point object at the depth z=15 cm.

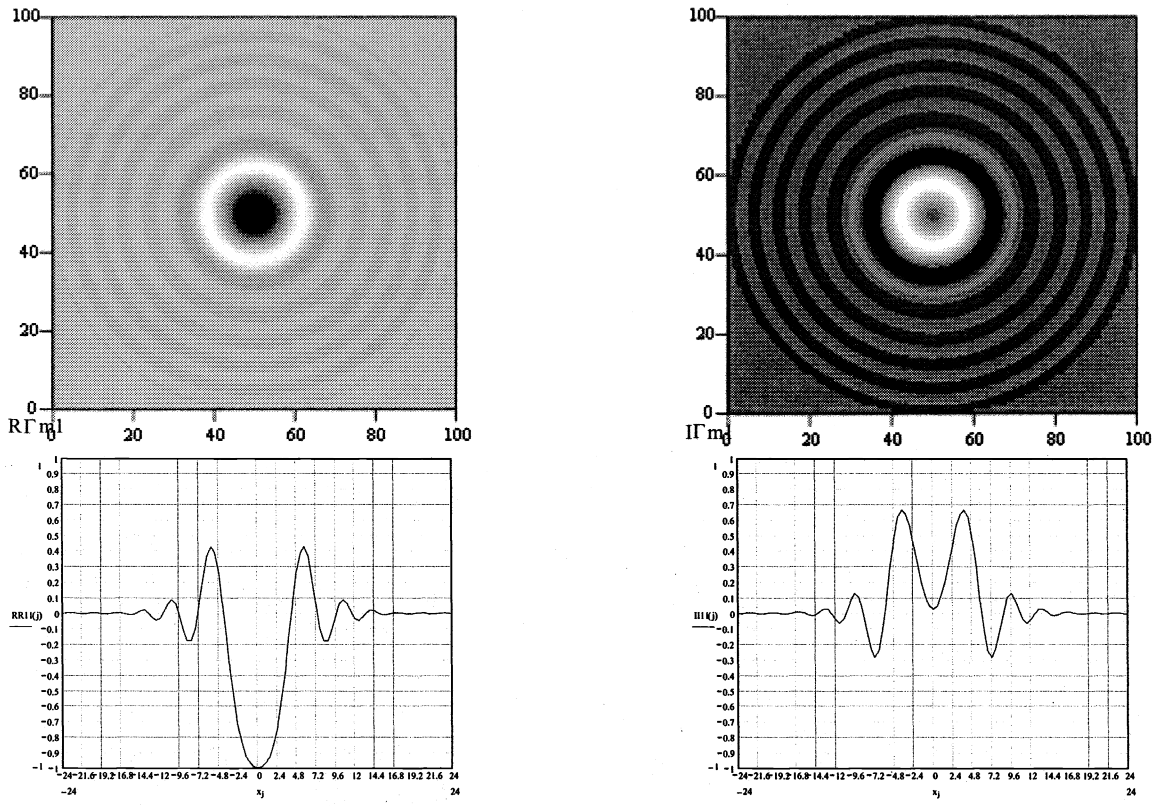

Figure 2. Quadrature components of the microwave hologram (eqn. 4) at D=6 cm,
f=3.7 GHz, ε_l=3.5, σ=10^{-2}, and depth of the object z =15 cm.

RECONSTRUCTION ALGORITHMS FOR COMPLEX AND AMPLITUDE SINGLE-FREQUENCY MICROWAVE HOLOGRAMS

Reconstruction of microwave holograms is similar to optimum processing of time-dependent signals (Shirman, 1970), when instead of time t spatial variable (x, y) in the recording plane of holographic signal is used.

The complex and amplitude holograms in eqns. 4 and 6 have differences in reception and processing methods because of presence of the unknown phases φ_T and φ_o. Dependence or independence of hologram energy in coordinates (x, y) from essential parameters of the holograms (x_T, y_T, z_T) and from non-essential interfering parameters φ_T and φ is important for the theory under investigation. For the single object with expected coordinates (x_o, y_o, z_o), complex and amplitude holograms energies in eqns. 4 and 6 can be written as

$$e_\kappa(z_0) = \frac{a_T^2}{2} \iint A^2\left(\sqrt{x^2+y^2}; z_0\right) dx\ dy\ , \tag{7}$$

$$\begin{aligned} e_a(z_0,\varphi) &= 4 a_T^2\, a_0^2 \\ &\times \iint A^2\left(\sqrt{x^2+y^2}; z_0\right) \cos^2\left\{\Phi\left(\sqrt{x^2+y^2}; z_0\right) + \varphi\right\} dxdy\ . \end{aligned} \tag{8}$$

These equations show that the displacement x_o and y_o and interfering parameter φ_T don't affect the complex hologram energy. The depth of the object is an energy influential parameter; as far as it influences width of the hologram main lobe on coordinates (x, y), see Fig. 2. This also confirms analogy with the optimum detection theory of the radar impulse signals. In case of the amplitude hologram the phase φ also becomes the energy influential parameter, which is caused by the absence of the second quadrature component.

Quadrature Coherent Receiver

For the case of quadrature coherent reception, we choose the complex hologram of the point object (eqn. 4) with expected coordinates (x_o, y_o, z_o) at the zero phase shift of signal reflection from the object as the support function for reconstruction procedure

$$\begin{aligned} \dot{S}(x-x_0, y-y_0; z_0) &= A\left(\sqrt{(x-x_0)^2+(y-y_0)^2}\ ; z_0\right) \\ &\times \exp\left\{ j\,\Phi\left(\sqrt{(x-x_0)^2+(y-y_0)^2}\ ; z_0\right)\right\}\ . \end{aligned} \tag{9}$$

For the case of complex hologram with the unknown phase φ_T, on the basis of analogy to time-dependent signal with the unknown phase (Shirman, 1970) the likelihood equation can be described as

$$\Lambda(x_0, y_0, z_0) = \exp\left\{-\frac{e_k(z_0)}{N_0}\right\} I_0\left(\frac{2}{N_0} Q(x_0, y_0, z_0)\right), \tag{10}$$

where $I_o(p)$ is modified Bessel's function and

$$Q(x_0, y_0, z_0) = \left|\dot{Q}(x_0, y_0, z_0)\right| = \left|\frac{1}{2}\iint \dot{U}(x, y)\,\dot{S}^*(x - x_0, y - y_0; z_0)\,dx\,dy\right| \quad (11)$$

is modulus of complex correlation integral.

Amplitude Receiver

If we consider the case of amplitude receiver, we assume that condition $a_T / a_0 << 1$ is valid. In this case, it is possible to disregard the second term $\left|\dot{U}(x, y)\right|^2$ in eqn. 5. Then as support function for hologram reconstruction we choose the amplitude hologram of the point object (eqn. 6) with coordinates (x_o, y_o, z_o)

$$S_\varphi(x - x_0, y - y_0; z_0) = A\left(\sqrt{(x - x_0)^2 + (y - y_0)^2}\,; z_0\right) \times\cos\left\{\Phi\left(\sqrt{(x - x_0)^2 + (y - y_0)^2}\,; z_0\right) + \varphi\right\}. \quad (12)$$

For the amplitude hologram $h(x, y)$ of the object with the known phase φ at the background of spatially non-correlated noise $\dot{N}(x, y)$ with spectral density N_o by analogy to the theory of time-dependent signal with the known phase (Shirman, 1970) the likelihood equation can be written as follows

$$\Lambda(x_0, y_0, z_0, \varphi) = \exp\left\{-\frac{e_a(z_0, \varphi)}{N_0} + \frac{2}{N_0} Q_\varphi(x_0, y_0, z_0)\right\}, \quad (13)$$

where

$$Q_\varphi(x_0, y_0, z_0) = \iint h(x, y) S_\varphi(x - x_0, y - y_0; z_0)\,dx\,dy \quad (14)$$

and the energy of the amplitude hologram is calculated according to eqn. 8. In order to get an optimal algorithm of hologram reconstruction the averaging of likelihood equation should be carried out in view of energy dependence on the signal phase

$$\Lambda(x_0, y_0, z_0) = \frac{1}{2\pi}\int_0^{2\pi} \Lambda(x_0, y_0, z_0; \varphi)\,d\varphi\;. \quad (15)$$

If the object depth is in the Fresnel zone of the antenna, the energy with the increase of z_o becomes asymptotic to $e_k(z_o)/2$, which is not depended on phase φ. Therefore this likelihood equation takes the regular form of eqns. 10 and 11 with two differences. Firstly, the average energy on both quadratures of the expected hologram, which is equal to half of the complex hologram energy, is used as the amplitude hologram energy. Secondly, the amplitude hologram is used in the correlation integral modulus (eqn. 11) instead of the complex hologram, i.e. $\dot{U}(x, y) = h(x, y)$,

$$\Lambda(x_0, y_0, z_0) \approx \exp\left\{-\frac{e_k(z_0)}{2N_0}\right\} I_0\left(\frac{2}{N_0} Q(x_0, y_0, z_0)\right), \quad (16)$$

$$Q(x_0, y_0, z_0) = \left|\iint h(x, y)\dot{S}^*(x - x_0, y - y_0; z_0)\,dx\,dy\right|. \quad (17)$$

In case of the coherent and amplitude receivers, the result of microwave hologram reconstruction can be presented by the functions $\Lambda(x_o, y_o, z_o)$ or $Q(x_o, y_o, z_o)$ constructed in three-dimensional space (x_o, y_o, z_o) or in its cross-sections. For the correlation integral, which is not being strictly optimal, a distortion of reconstruction results on variable z_o occurs. The reason is absence of the multiplier, which depends on energy of the hologram and is taken into account in case of the likelihood equation.

EXPERIMENTAL RESULTS

The packet, consisting of thirty plasterboards with dimensions 1.5 m by 1.5 m, was used as the test bed. The packet thickness was 40 cm. Different objects were placed between the boards. Objects under investigation consisted of two metal wires, bugging device placed between the wires, seven coins of 25 mm diameter under bugging device, and cavity in second plasterboard. The coins were placed at different depths in the packet. Depths of coins and cavity are depicted in the Table 1.

Table 1

Object Number	Object Type	Depth (cm)
1	coin	2.5
2	coin	5.0
3	coin	7.5
4	cavity	1.25
5	coin	10.0
6	coin	12.5
7	coin	15.0
8	coin	17.5

The amplitude microwave hologram of test bed at frequency of 3.7 GHz (see Fig. 3) was used to examine validity of the developed reconstruction models. In this case the correlation algorithm (eqn. 14) and algorithm of reconstruction, which is based on the likelihood equation (eqn. 13) with the known phase φ changing in range 0°-180°, were used. The phase value with best reconstruction result is considered as the closest to the actual value.

Hologram reconstruction (eqns. 13 and 14) with the known phase was carried out with the use of the support function of the point object (eqn. 12). In this algorithm we also used beforehand-tabulated dependences of the amplitude $A(\rho, z)$ and phase $\Phi(\rho, z)$, which were placed in matrixes with dimensions 101 on ρ and 21 on z with the appropriate step for these variables.

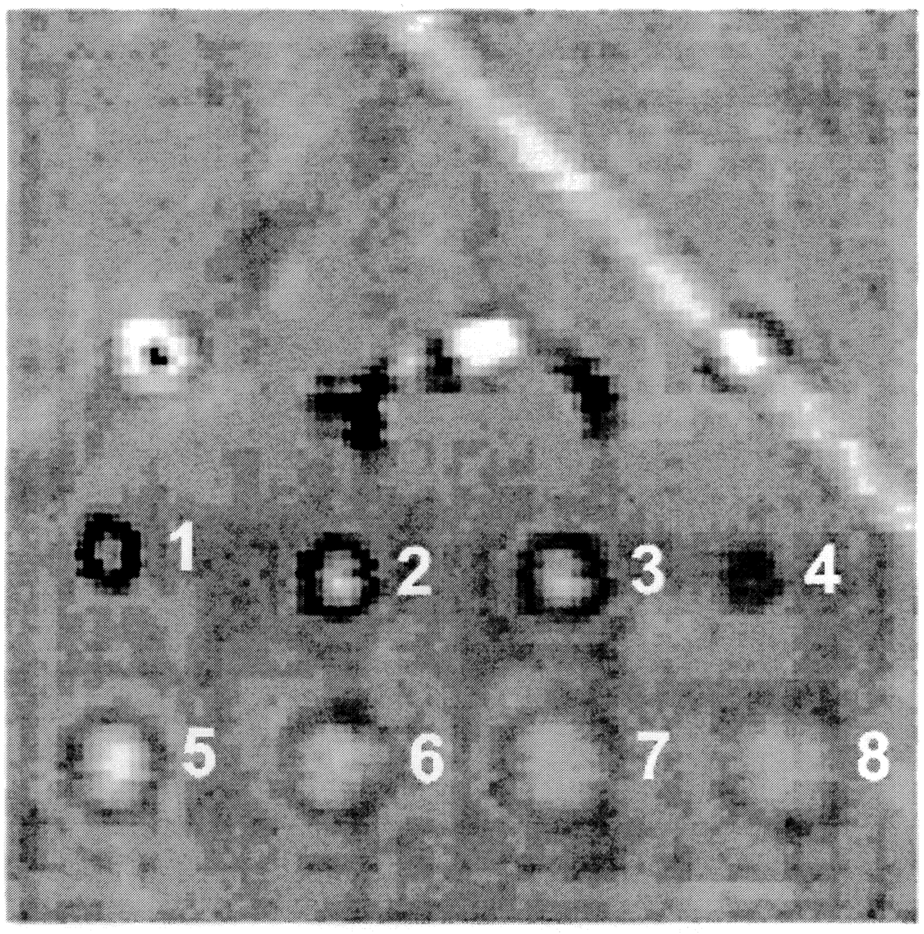

Figure 3. The microwave hologram of the test bed received by RASCAN radar at frequency of 3.7 GHz.

The results of reconstruction with the use of the correlation algorithm (eqn. 14) for eight selected objects with six phase values are submitted in a Fig. 4 in the form of gray images. At each phase value the images represent correlation integral modulus (eqn. 14) in two depth cross-sections, which are perpendicular to horizontal surface of the test bed and cross the objects centers on two lines. The left column of the images corresponds to shallowly buried objects 1-4, and the right column corresponds to deeper objects 5-8. Greater brightness on the images matches greater value of the correlation integral modulus. Judging by the examined images, it is clear that holograms reconstruction takes place and its efficiency is most brightly expressed for the deeper objects 5-8.

If phase value equals to 90°, the maximum brightness of the images corresponds to depths: 9.3 cm, 11.7 cm, 14.9 cm, and 17.4 cm, which coincides well with the actual depths of objects 5-8. If phase value doesn't equal to 90°, we have either absence of reconstructed image or erroneous value of depth. For objects 1 and 4 (the depths equal to 2.5 cm and 1.25 cm) it is practically impossible to reconstruct hologram at any value of initial phase. It is apparently a consequence of some discrepancy of the Fresnel-Kirchhoff scalar model with reality and replacement of the real antenna by the common-mode aperture antenna.

The sizes of reconstructed "spots" on a horizontal plane at zero depth don't surpass the antenna aperture diameter and on level 0.5 - 0.7 correspond to about half of the antenna diameter, i.e. 3 cm. This value is much less than the initial images diameters of objects 5-8 on the hologram (see Fig. 3).

We could acquire nearly the same results with the use of reconstruction algorithm on the basis of the likelihood equation (eqn. 13). In this case the estimation approaching to the actual values of an objects depths also corresponds to phase 90°. Reconstruction of the hologram is also possible for objects 2 and 3. Their depths errors amount to 1.5 and 2.5 cm, respectively. For deeper objects the errors are almost the same as for correlation algorithm. The main difference from the results in Fig. 4 is that the images change period for the correlation integral modulus depending on the phase is equal to 180°. While as the images change period for the likelihood equation, in which value of correlation integral is used, equals 360°.

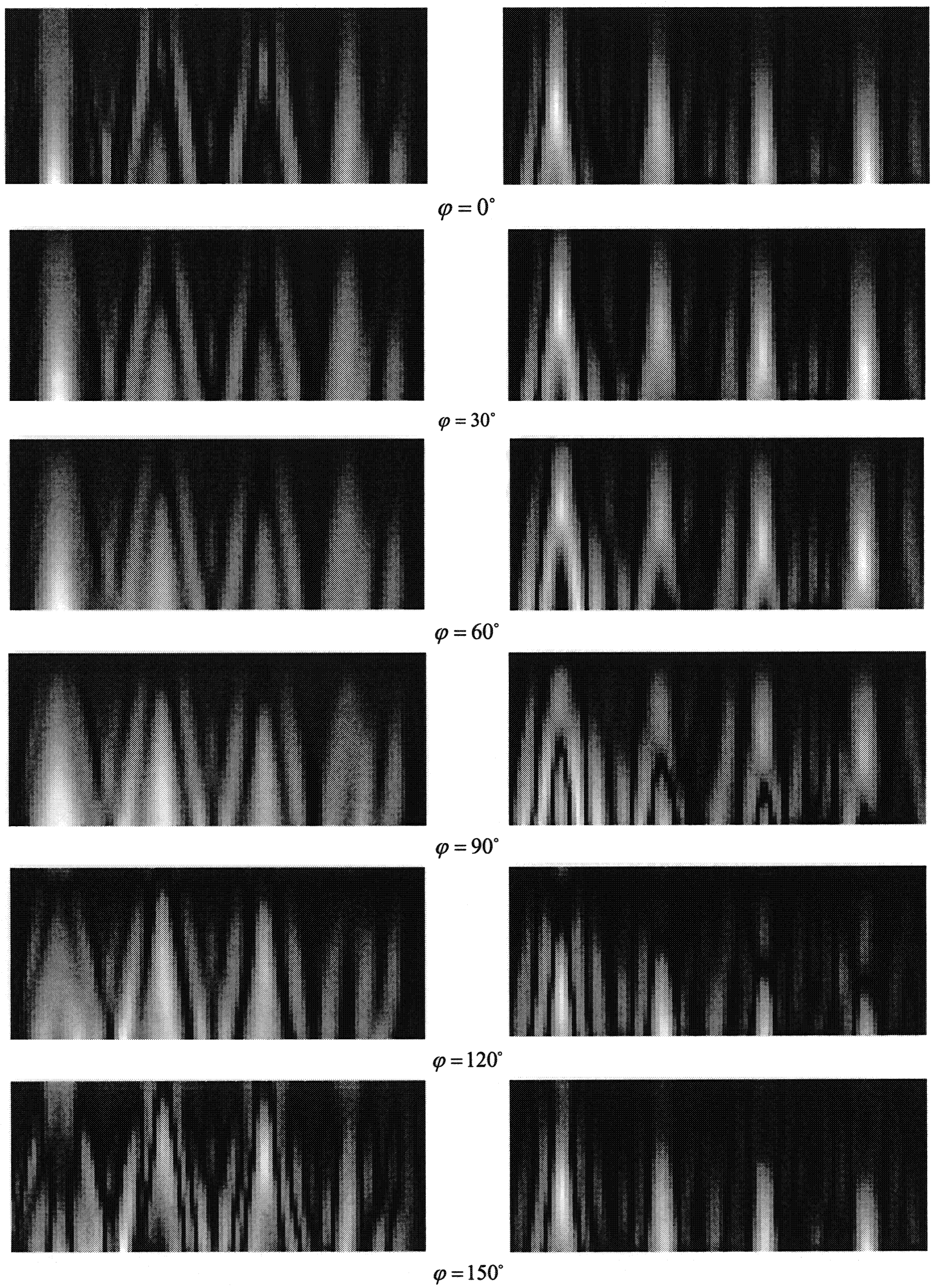

Figure 4. Holograms reconstruction in depth with the help of correlation algorithm with the known phase φ for objects 1-8.

The analysis of occurring changes in the images testifies that with the increase of phase φ in the reconstruction function (eqn. 12) bright spots, corresponding to the position of objects on depth, move upwards. Thus the position of the bright spots becomes closer to the actual depths of objects at the actual phase value. Further increase of the phase results in reduction of the spot brightness and increasing of its diameter with further displacement in the area of smaller depths and finally to destruction of the spot. The process lasts as long as the phase again doesn't reach the beginning of the next period. Similar periodic changes of the images with reconstruction function phase change obviously give more complete picture of synthesized holograms reconstruction and more correct and accurate estimation of object depth using correct criterion, than reconstruction on the basis of phase averaging in the likelihood equation (eqn. 16) or modulus of complex correlation integral (eqn. 17).

These conclusions are confirmed by the results of hologram reconstruction with the help of correlation algorithm (eqn. 17) for objects 1-8 (see Fig. 5), and also by similar results with the use of the algorithm according to eqn. 16. It can be seen that the depth resolution, which determines quality of hologram reconstruction, is better for algorithm with the known phase than for algorithm with the unknown phase according to eqn. 16.

Moreover the estimated value of depth, which corresponds to the maximal brightness of the image, is bigger than the actual one. For the known phase the estimated value of depth doesn't differ much from the exact value.

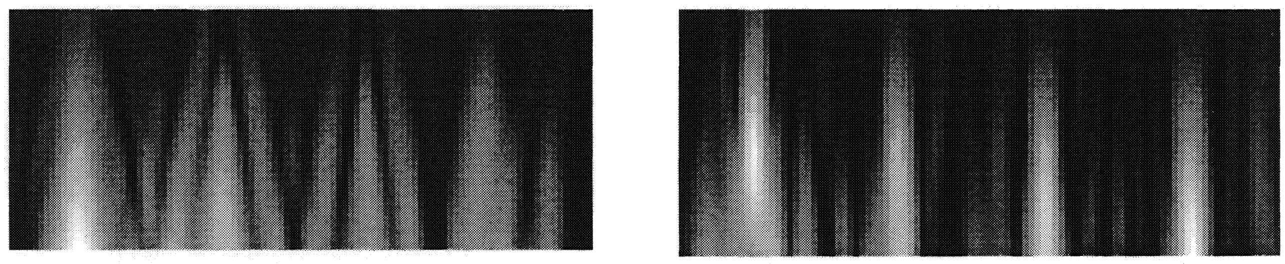

Figure 5. Application of the correlation algorithm with the unknown phase for hologram reconstruction of objects 1-8.

CONCLUSIONS

In this paper the algorithms of the holograms reconstruction, obtained by RASCAN radar, are proposed. The algorithms take into account phases of the reference signal and signal reflected from the subsurface point object. The proposed mathematical models are based on the use of the amplitude-phase characteristics of the near and reactive fields considered in scalar approximation with the help of the Fresnel-Kirchhoff integral equation for the circular antenna aperture.

The model allows to develop reconstruction method for the complex and amplitude single-frequency holograms synthesized at scanning of the RASCAN antenna on surface of lossy half-space with known permittivity. The method is analogous to optimum processing of time-dependent signals with known and unknown initial phase.

Reconstruction of the experimental microwave hologram with the use of the point object hologram as a support hologram confirms theoretical assumptions on depth resolution improvement for monochromatic mode operation of the subsurface radar considered. The further efforts and research will be devoted to the reconstruction algorithms of the radar multi-frequency holograms.

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